Exam. Code : 103205
Subject Code : 8059

B.A./B.Sc. 5th Semester (Old Syllabus 2016)

MATHEMATICS

Paper—II (Linear Algebra)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **five** questions in all, selecting at least two questions from each Section.

SECTION—A

I. (a) Prove that <Q, $*$> where Q is the set of all rationals except 1, is an abelian group under binary operation $*$ as defined as $a * b = a + b - ab$.

(b) Prove that the set Z of integers is a commutative ring with respect to usual addition and multiplication of integers. 5,5

II. (a) If W_1 and W_2 are any two subspaces of a vector space V(F), prove that $W_1+W_2 = \{x+y : x \in W_1, y \in W_2\}$ is a subspace of V(F).

(b) Let $v_1 = (1, 2, -1)$; $v_2 = (2, -3, 2)$; $v_3 = (4, 1, 3)$ and $v_4 = (-3, 1, 2)$ be the vectors in $\mathbb{R}^3(\mathbb{R})$, show that $L(\{v_1, v_2\}) \neq L(\{v_3, v_4\})$. 5,5

III. (a) If V(F) be a vector space, prove that the set S of non-zero vectors, $v_1, v_2, \ldots v_n \in V$ is L.D. iff some element of S is a linear combination of others.

(b) If v_1, v_2, v_3 are linearly independent vectors of V(F), show that the vectors $v_1 + v_2, v_2 + v_3, v_3 + v_1$ are L.I. 5,5

IV. (a) If U and W are two subspaces of a finite dimensional vector space V(F), prove that $\dim(U + W) = \dim U + \dim W - \dim(U \cap W)$.

(b) Extend $\{(3, -1, 2)\}$ to two different bases of $\mathbb{R}^3$. 6,4

V. (a) Let W_1 and W_2 be the subspaces of $\mathbb{R}^3(\mathbb{R})$, where

$W_1 = \{(a, b, c) : b = 2a, c = a+b\}$

$W_2 = \{(a, b, c) : 2a + b - 3c = 0\}$

Find a basis and dimension of :—

(i) W_1 (ii) W_2

(b) Find a basis and dimension of the solution space S of the system of equations : $x + 2y - 4z + 3s - t = 0$,

$x + 2y - 2z + 2s + t = 0,\ 2x + 4y - 2z + 3s + 4t = 0.$ 5,5

SECTION—B

VI. (a) Find T(x, y, z) where $T : \mathbb{R}^3 \to \mathbb{R}$ is defined by $T(1, -1, 1) = 3$, $T(0, 1, -2) = 1$, $T(0, 0, 1) = -2$.

(b) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be a L.T. defined as $T(x, y) = (x + y, x - y, y)$.

Verify Rank (T) + Nullity(T) = dim $\mathbb{R}^2$. 4,6

VII. (a) Find a linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^4$ whose range space is generated by (1, 2, 0, –4) and (2, 0,–1, –3).

(b) Give an example of two linear transformations T_1 and T_2 such that $T_1T_2 \neq T_2T_1$. 6,4

VIII. (a) Prove that a linear transformation $T : V \to W$ is non-singular, iff the set of images of L.T. set is L.I.

(b) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be defined by

$T(x, y, z) = (2x, 4x-y, 2x + 3y - z)$

Show that T is invertible and find T^{-1}. 5,5

IX. (a) Let V(F) and W(F) be finite dimensional vector spaces over the same field F and $T : V \to W$ be a L.T. If B_1 and B_2 be the ordered basis of V and W respectively, prove that for any vector $v \in V$,

$$[T; B_1, B_2]\,[v; B_1] = [T(v); B_2].$$

(b) Let V be a V.S. of all functions $f: \mathbb{R} \to \mathbb{R}$. Define a differential operator D on V as $D(f) = \frac{df}{dt}\ \forall f \in V$. Find the matrix of D w.r.t. basis $B = \{1, t, \sin 3t, \cos 3t\}$.

5,5

X. (a) Find the matrix representation of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ defined by $T(x,y) = (3x-2y, 0, x+4y)$ w.r.t. ordered bases. $B_1 = \{(1, 1), (0, 2)\}$ and $B_2 = \{(1,1,0), (1,0,1), (0,1,1)\}$ for $\mathbb{R}^2$ and $\mathbb{R}^3$ respectively.

(b) If the matrix of a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ relative to usual basis, is $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & -1 \\ -1 & -1 & 0 \end{bmatrix}$. Find the matrix of T relative to the basis $B_1 = \{(0,1,-1), (-1, 1, 0), (1, -1, 1)\}$. 5,5